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Limit on $Br(b \to sg)$ in Two Higgs Doublet Models

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Abstract

Using the recent CLEO measurement of $Br(b \to s\gamma)$, we find that the branching ratio of $b \to sg$ cannot be larger than 10% in two Higgs doublet models. The small experimental value of $Br(b \to e\bar{\nu}X)$ can no longer be explained by charged Higgs boson effects.

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It is well known that the process $b \to s\gamma$ is extremely sensitive to new physics beyond the standard model, in particular, that containing a charged Higgs boson. In 1993, the CLEO collaboration placed an upper limit of $Br(b \to s\gamma) < 5.4 \times 10^{-4}$ on the inclusive branching ratio [1], which has inspired a large number of studies of this decay in various models for new physics [2]. Stringent constraints are obtained. Recently, CLEO has *measured* the inclusive branching ratio to be [3] $(2.32 \pm 0.51 \pm 0.29 \pm 0.32) \times 10^{-4}$, in short, $(2.3 \pm 0.7) \times 10^{-4}$, corresponding to the 95% confidence level range of

$$1 \times 10^{-4} < Br(b \to s\gamma) < 4 \times 10^{-4}. \tag{1}$$

In the standard model, there is a surprisingly large QCD enhancement of $b \to s\gamma$ [4] amplitude. This has stimulated intense efforts in calculating QCD corrections to leading order (LO), as well as partial calculations to next-to-leading order (NLO) [5,6]. The CLEO result of eq. (1) is not far from the prediction of $Br(b \to s\gamma)$ in the standard model. This implies that not much room is left for new physics contributions to other $b \to s$ transitions such as $b \to sg$, where the emitted gluon is "on-shell".

It was shown in ref. [7] that for some choices of parameters in two Higgs doublet models, charged Higgs boson effects may enhance the decay branching ratio of $b \to sg$ beyond the 10% level. Grządkowski and Hou [8] have pointed out that if $b \to sg$ rate is at the $(10-20)\%$ level, the discrepancy on $Br(b \to e\nu X)$ between experimental measurement $(10.7 \pm .5\%)$ [9] and theoretical expectations ($> 12\%$ in the standard model) [10] could be resolved. It is therefore of interest to check whether the possibility of $Br(b \to sg) \sim 10\%$ still holds once one includes the constraint imposed by eq. (1).

In this report we focus on two Higgs doublet models (2HDM). These models are very simple extensions of the standard model, yet they exhibit some of the characteristics of a more complicated scalar structure typical of most theories beyond the standard model. We will consider the two distinct models (I and II) that naturally avoid tree-level FCNCs [11]. In Model I, one doublet (ϕ_2) couples to all fermions and the other (ϕ_1) decouples from the fermion sector. In Model II, ϕ_2 couples to up-type quarks while ϕ_1 couples to

down-type quarks. This type of model occurs in minimal realization of supersymmetry, or in models with a Peccei-Quinn symmetry [12]. The major non-standard feature of these models is the appearance of extra physical scalar fields. We consider only the effect of charged Higgs bosons. Two parameters are sufficient to account for the additional effects. We take these to be m_H, the mass of the charged Higgs boson, and $\xi \equiv v_1/v_2$, the ratio of the vacuum expectation values of ϕ_1 and ϕ_2. Note that $\xi = 1/\tan\beta$, as is commonly used in supersymmetric models. In 2HDM, the charged Higgs couple to quarks with the same quark mixing matrix as the standard charged current.

The standard model calculation for $b \to s\gamma$, including up to date QCD corrections, can be found, for example, in Refs. [5,6]. The inclusive branching ratio is given by

$$\begin{aligned} Br(b \to s\gamma) &= \frac{\Gamma(b \to s\gamma)}{\Gamma(b \to ce\bar{\nu})} \, Br(b \to ce\bar{\nu}) \\ &= \frac{|V_{ts}^* V_{tb}|^2}{|V_{cb}|^2} \frac{6\alpha}{\pi f(m_c/m_b)} \, \frac{1}{\Omega(m_t/M_W, \mu)} \, |C_7^{eff}(\mu)|^2 \, Br(b \to ce\bar{\nu}), \end{aligned} \tag{2}$$

where the Wilson coefficient

$$C_7^{eff}(\mu) = \eta^{\frac{16}{23}} C_7(M_W) + \frac{8}{3}\left(\eta^{\frac{14}{23}} - \eta^{\frac{16}{23}}\right) C_8(M_W) + C_2(M_W) \sum_{i=1}^{8} a_i \eta^{b_i}, \tag{3}$$

includes short distance effects at M_W scale, while perturbative QCD effects are accumulated when running down to the physical scale μ, with $\eta = \alpha_S(M_W)/\alpha_S(\mu)$. In eq. (2), the phase space factor $f(z)$ is given by

$$f(z) = 1 - 8z^2 + 8z^6 - z^8 - 24z^4 \ln z, \tag{4}$$

and the quantity $\Omega(z)$ contains the $O(\alpha_S)$ QCD corrections to the semileptonic decay rate [13,14] and is given by

$$\Omega(x, \mu) \simeq 1 - \frac{2\alpha_S(\mu)}{3\pi} \left[\left(\pi^2 - \frac{31}{4}\right)(1-x)^2 + \frac{3}{2}\right]. \tag{5}$$

The scheme-independent numbers a_i and b_i are given by [15]

$$\begin{aligned} a_i &= \left(\tfrac{626126}{272277}, \quad -\tfrac{56281}{51730}, \quad -\tfrac{3}{7}, \quad -\tfrac{1}{14}, \quad -0.6494, \quad -0.0380, \quad -0.0186, \quad -0.0057\right), \\ b_i &= \left(\tfrac{14}{23}, \quad \tfrac{16}{23}, \quad \tfrac{6}{23}, \quad -\tfrac{12}{23}, \quad 0.4086, \quad -0.4230, \quad -0.8994, \quad 0.1456\right), \end{aligned} \tag{6}$$

respectively. Defining $x = m_t^2/M_W^2$, $h = m_t^2/M_H^2$, the coefficients $C_i(M_W)$ are [7]

$$
\begin{aligned}
C_2(M_W)^I &= 1, \\
C_7(M_W)^I &= -\frac{1}{2}A(x) + \xi^2\left[B(h) - \frac{1}{6}A(h)\right], \\
C_8(M_W)^I &= -\frac{1}{2}D(x) + \xi^2\left[E(h) - \frac{1}{6}D(h)\right],
\end{aligned}
\tag{7}
$$

for Model I, and [7,16]

$$
\begin{aligned}
C_2(M_W)^{II} &= 1, \\
C_7(M_W)^{II} &= -\frac{1}{2}A(x) - B(h) - \frac{1}{6}\xi^2 A(h), \\
C_8(M_W)^{II} &= -\frac{1}{2}D(x) - E(h) - \frac{1}{6}\xi^2 D(h),
\end{aligned}
\tag{8}
$$

for Model II, where

$$
\begin{aligned}
A(x) &= \frac{-x}{12(1-x)^4}\left[6x(3x-2)\ln x + (1-x)(8x^2+5x-7)\right], \\
D(x) &= \frac{x}{4(1-x)^4}\left[6x\ln x - (1-x)(x^2-5x-2)\right], \\
B(x) &= \frac{x}{12(1-x)^3}\left[(6x-4)\ln x + (1-x)(5x-3)\right], \\
E(x) &= \frac{-x}{4(1-x)^3}\left[2\ln x + (1-x)((3-x)\right].
\end{aligned}
\tag{9}
$$

Analogously, the branching ratio for $b \to sg$, where the gluon is *on-shell* (in the sense of a "gluon jet"), can be written as,

$$
Br(b \to sg) = \frac{|V_{ts}^* V_{tb}|^2}{|V_{cb}|^2}\frac{8\alpha_S(\mu)}{\pi f(m_c/m_b)}\frac{1}{\Omega(m_t/M_W,\mu)}\,|C_8^{eff}(\mu)|^2\,Br(b \to ce\bar{\nu}),
\tag{10}
$$

where [5]

$$
\begin{aligned}
C_8^{eff}(\mu) = &\left[C_8(M_W) + \frac{313063}{363036}\right]\eta^{\frac{14}{23}} - 0.9135\eta^{0.4086} \\
&+0.0873\eta^{-0.4230} - 0.0571\eta^{-0.8994} - 0.0209\eta^{0.1456}.
\end{aligned}
\tag{11}
$$

Notice the explicit μ-dependence of eq. (10) on $\alpha_S(\mu)$. The μ scale of this α_S does not have to be the same as that of $C_8^{eff}(\mu)$, but we treat them as if they are the same. From eqs. (2) and (10), we form the ratio

$$R \equiv \frac{Br(b \to sg)}{Br(b \to s\gamma)} = \frac{4}{3} \frac{\alpha_S(\mu)}{\alpha} \left| \frac{C_8^{eff}(\mu)}{C_7^{eff}(\mu)} \right|^2 , \tag{12}$$

which is independent of m_c/m_b.

The scale μ denotes the renormalization scale of the effective $b \to s\gamma$ Hamiltonian. It should be of order m_b, but need not be exactly equal to m_b, and we shall take it to be in the range between 2.5 to 10 GeV as used in Ref. [5,17]. For simplicity we have taken it to be the same as the scale at which the parameter α_S is expanded for the QCD corrections to the semileptonic decay rate, eq. (5). We express $\alpha_S(\mu)$ in terms of its value at $\mu = M_Z$, i.e.

$$\frac{\alpha_S(M_Z)}{\alpha_S(\mu)} = 1 - \beta_0 \frac{\alpha_S(M_Z)}{2\pi} \ln\left(\frac{M_Z}{\mu}\right) , \tag{13}$$

in the leading logarithmic approximation where $\beta_0 = 11 - \frac{2}{3}N_f = 23/3$ ($N_f = 5$).

In calculating the branching ratios of $b \to s\gamma$ and $b \to sg$ in eqs. (2) and (10), one needs to know the values of the scale μ and the ratio m_c/m_b, which are not well determined. However, since in our case we only want to find out the maximum value of $Br(b \to sg)$ that is still allowed, and since the ratio R does not depend on m_c/m_b, we keep m_c/m_b at some fixed value $\sim 1/3$. Similarly, we neglect the uncertainties arising from the CKM mixing elements in our calculations of the $b \to s$ decay branching ratios. The main uncertainty is therefore in the scale μ.

We study $b \to sg$ and $b \to s\gamma$ numerically for different sets of parameters ξ and M_H. We find that a smaller value of μ gives largest $Br(b \to sg)$. This is demonstrated in Fig. 1 with $\xi = 1\,,\ 2$ and $M_H = m_t = 170\ GeV$. In Figs. 2(a) and (b), we present the branching ratio of $b \to sg$ decay for $m_t = 170$ GeV and $\mu = 2.5$ GeV for Models I and II, respectively. The hatched region to the right is ruled out by the CLEO upper bound on $Br(b \to s\gamma)$ of eq. (1). We notice that the CLEO limit has excluded most of the parameter space in the $\xi - M_H$ plane for Model II. The lower bound of $Br(b \to s\gamma) > 1.0 \times 10^{-4}$ excludes the second hatched region to the left in Fig. 2(a) for Model I. We further overlay (shaded) the combined constraints on CKM mixing matrix (since m_t is fixed here at 170 GeV) from ϵ parameter in $K \to \pi\pi$ decay, B-$\bar{B}$ mixing, and the ratios $|V_{cb}/V_{us}|$ and $|V_{ub}/V_{cb}|$ [18]. This constraint

is rather stringent for heavy top, and is the same for both Model I and II since the top coupling is common in both models. The solid, dot and dash curves in Fig. 2(a) represent $Br(b \to sg)$ being 0.1% (< 0.1% between the two solid lines), 1% and 8%, respectively in Model I. For sake of illustration, however, for Model II the corresponding lines in Fig. 2(b) are for $Br(b \to sg) = 0.7\%$, 1.0% and 1.5%, respectively. From Fig. 2 we see that, if the top is heavy as suggested by recent observation of CDF [19], $Br(b \to sg)$ can at most be of order 1% for both Model I and II. For Model I, in fact, it would be rather difficult to go much beyond 0.1%.

In Fig. 2(a), we find that the branching ratio of $b \to sg$ goes to zero inside the two solid lines due to the destructive interference between the charged Higgs and the standard model contributions. $Br(b \to sg)$ in the region to the left of the solid line increases and reaches the standard model value ($\sim 4.5 \times 10^{-3}$) when $\xi = 0$. Note that in the $\xi - M_H$ plane, for Model I, the interesting region allowed by the CLEO data is more or less ruled out by the CKM constraints, while for Model II, the variation of $Br(b \to s\gamma)$ is much faster than that of $Br(b \to sg)$. Thus, in either case, new improvements on the experimental measurement of the $b \to s\gamma$ decay will imply no significant changes on the limits of $Br(b \to sg)$.

An intriguing possibility still exists for $M_H + m_b < m_t < M_W + m_b$ [20], allowed by all known constraints, including $b \to s\gamma$ and B-$\bar{B}$ mixing. When top is light, substantial charged Higgs contributions may in fact be called for. If one takes the heavy quark production signal observed by CDF [19] seriously, it may actually be the fourth generation t' quark. In that case, all loop effects are subject to GIM cancellation, and can be made ineffective. The crucial point, however, is that $t \to bH^+$ overwhelms $t \to bW^*$ in this domain, and can allow the top quark to elude past searches at hadronic colliders. For Model I, assuming that the heavy quark seemingly observed by CDF [19] does not dominate in the loop processes, we find that this can happen for $\xi = 1/\tan\beta \sim 2$. We illustrate in Fig. 3(a) the allowed region for $Br(b \to sg)$ for $M_H < m_t = 70$ GeV. Although there is still no large enhancement, the CKM constraint is now more forgiving, and a 3% branching ratio for $b \to sg$ is possible. This cannot be considered small when compared with the standard model expectation of

order 10^{-3} [6]. For Model II, $b \to s\gamma$ provides a very stringent constraint, and in particular it is difficult to evade the direct search for $t \to bH^+ \to b\tau^+\nu$ [21]. Nevertheless, combining the two constraints, it is found [22] that the region $\xi = 1/\tan\beta \sim 1$ is allowed for having a light top decaying via charged Higgs (which does not decay dominantly via $\tau\nu$), especially when one takes into account all the possible sources of errors in making estimates. We plot in Fig. 3(b), with same notation as in Fig. 2(b), the expected $Br(b \to sg)$ that may still be allowed for Model II. The solid line corresponding to $Br(b \to sg) = 0.7\%$ now falls outside of the figure. We find that the maximum value of $Br(b \to sg)$ is about 0.9%, which is indeed smaller than the case for Model I.

As stressed in ref. [8], $b \to sg$ at the 10% level or higher could account for the apparent discrepancy on $Br(b \to e\nu{+}X)$ between experiment and theory. We find that this possibility is quite definitely ruled out, by the combined limits of $b \to s\gamma$ and CKM matrix, especially if one takes the CDF heavy quark production signal as due to the top quark. However, in case the top is actually light (and CDF signal is either faked or due to *new* heavy quarks), $b \to sg$ could still be at 3.5% level. Since the other possibility for suppressing $Br(b \to e\nu + X)$ by having $b \to \tau\nu + X$ at 10% level or higher is also ruled out by ALEPH collaboration [23], the two Higgs doublet models cannot help alleviate the inclusive semileptonic b decay problem. Perhaps one would have to opt for large $\alpha_S(M_Z)$ (of order 0.13) and a low μ scale (*e.g.* $\mu \sim m_b/2$) for B decay processes, as suggested by Altarelli and Petrarca [24]. This would imply that $\alpha_S > 0.3$ for B decay processes.

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FIGURES

The ratio R as a function of the scale μ in (a) Model I and (b) Model II. The solid and dash curves are for $\xi = 2$ and 1, respectively.

Branching ratio of $b \to sg$ decay for $m_t = 170$ GeV and $\mu = 2.5$ GeV for (a) Model I and (b) Model II. The hatched region to the right is ruled out by the CLEO upper bound on $Br(b \to s\gamma)$, while a second region to the left in (a) is excluded by imposing a lower bound of $Br(b \to s\gamma) > 1.0 \times 10^{-4}$. The shaded region is forbidden by constraints on the CKM matrix. The solid, dot and dash curves represent $Br(b \to sg)$ being 0.1% (< 0.1% between the two solid lines), 1% and 8% in (a), while in (b) they correspond to $Br(b \to sg) = 0.95\%\,,\ 1.0\%$ and 1.3%, respectively.

Same as Fig. 2 but with $m_t = 70\ GeV$.